SSP Exercise3

Dispersion relation: $\varepsilon_F = \dfrac{\hbar^2 k_F{}^2}{2m}$

Number of electrons in Fermi sphere $= 2\left(\dfrac{4\pi k_F^3}{3}\right)\left(\dfrac{V}{8\pi^3}\right) = \dfrac{k_F^3}{3\pi^2}V$

2 spin orientations

k-space volume

V=real space volume

$\Rightarrow$ Electron density, $n = \dfrac{k_F^3}{3\pi^2}$

use dispersion relation to eliminate k_F , $\boxed{\varepsilon_F = \dfrac{\hbar^2 k_F{}^2}{2m} = \dfrac{\hbar^2}{2m}\left(3\pi^2 n\right)^{\frac{2}{3}}}$ **[7]**

(i) Fcc → 4 atom/unit cell → $n=4/a^3$= 8.5 x 10^{28} m^{-3}, put into above equation→

$\boxed{\varepsilon_F = 6.99 \text{ eV}}$ **[6]**

(ii) Number of electrons in Fermi disc, $N = 2\left(\pi k_F^2\right)\left(\dfrac{A}{4\pi^2}\right) = \dfrac{k_F^2}{2\pi}A$

k-space area

A=real space area

Electron density, $n = \dfrac{N}{A} = \dfrac{k_F^2}{2\pi} \Rightarrow \boxed{\varepsilon_F = \dfrac{\hbar^2 k_F{}^2}{2m} = \dfrac{\hbar^2}{2m}\left(2\pi n\right) = 0.7 \text{ meV}}$ **[6]**

(iii) Electrons density = 1/(0.8 x 10^{-9}) = 1.25 x 10^{9} m^{-1}

Number of electrons in Fermi length = $2\left(2k_F\right)\left(\dfrac{L}{2\pi}\right) = \dfrac{2k_F}{\pi}L$

k-space length

L=real space length

Electron density, $n = \dfrac{N}{L} = \dfrac{2k_F}{\pi} \Rightarrow \boxed{\varepsilon_F = \dfrac{\hbar^2 k_F{}^2}{2m} = \dfrac{\hbar^2}{2m}\left(\dfrac{\pi n}{2}\right)^2 = 146 \text{ meV}}$ **[6]**